



ΘΕΜΑ Α

A1 γ

A2 α

A3 γ

A4 δ

A5 α Σ β Λ γ Σ δ Σ ε Λ

ΘΕΜΑ Β

B₁. Απάντηση iii)

απόδειξη

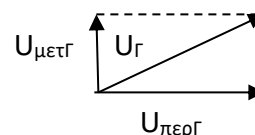
$$\kappa.\chi.o. \quad \vec{v}_{max} = \vec{0} \Leftrightarrow \vec{v}_{μετ} + \vec{v}_{περ} = \vec{0} \Leftrightarrow \vec{v}_{μετ} = -\vec{v}_{περ} \Leftrightarrow |\vec{v}_{μετ}| = |\vec{v}_{περ}| \Leftrightarrow V_{cm} = \omega R$$

$$\text{σημείο Α} \quad \vec{v}_A = \vec{v}_{μετ_A} + \vec{v}_{περ_A} \Leftrightarrow |\vec{v}_A| = v_{cm} + \omega R = 2v_{em}$$

$$\text{σημείο Γ} \quad \vec{v}_\Gamma = \vec{v}_{μετ_\Gamma} + \vec{v}_{περ_\Gamma} \Leftrightarrow \vec{v}_\Gamma = \vec{v}_{μετ_\Gamma} + \vec{v}_{περ_\Gamma} \Leftrightarrow |\vec{v}_\Gamma| = \sqrt{v_{cm}^2 - \left(\omega \frac{R}{2}\right)^2} = \sqrt{v_{cm}^2 + \frac{v_{cm}^2}{4}} =$$

$$\sqrt{\frac{5v_{cm}^2}{4}} = \frac{\sqrt{5}}{2} v_{cm}$$

$$\text{Άρα} \frac{|\vec{v}_\Gamma|}{|\vec{v}_A|} = \frac{\frac{\sqrt{5}}{2} v_{cm}}{2v_{em}} = \frac{\sqrt{5}}{4}$$



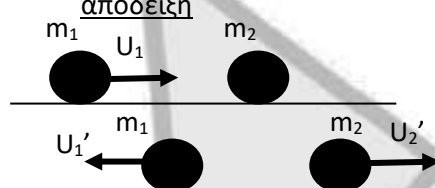
B₂

Απάντηση ii)

απόδειξη

1^η περίπτωση

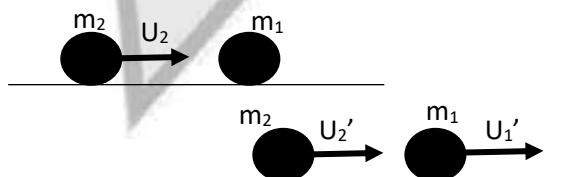
$$v_2' = \frac{am_1}{m_1 + m_2} v_1$$



$$\Pi_1 = \frac{K_2'}{K_1} 100\% = \frac{\frac{1}{2}m_2 v_2'^2}{\frac{1}{2}m_1 v_1^2} 100\% = \frac{m_2 \frac{4m_2^2}{(m_1 + m_2)^2} v_1^2}{m_1 v_1^2} 100\% = \frac{4m_1 m_2}{(m_1 + m_2)^2} 100\%$$

2^η περίπτωση

$$v_1' = \frac{2m_2}{m_1 + m_2} v_2$$



$$\Pi_2 = \frac{K_1'}{K_2} 100\% \Leftrightarrow \Pi_2 = \frac{\frac{1}{2}m_1 \frac{4m_2^2}{(m_1 + m_2)^2} v_2^2}{\frac{1}{2}m_2 v_2^2} 100\% = \frac{4m_1 m_2}{(m_1 + m_2)^2} 100\% \quad \text{Άρα } \Pi_1 = \Pi_2$$



B₃

Απάντηση i)

απόδειξη

Εξίσωση Bernoulli

Εξ. Επιφάνεια , 0

$$P_{ατμ} + \frac{1}{2} \rho v^2 + \rho g H = P_{ατμ} + \frac{1}{2} \rho v_1^2 + \rho g h_1 \Leftrightarrow \frac{1}{2} \rho v_1^2 = \rho g (H - h_1) \Leftrightarrow v_1^2 = 2g(H - h_1) \quad (1)$$

Οριζόντια βολή

$$\begin{cases} s = v_1 \cdot t_{\pi\tau} \Leftrightarrow t_{\pi\tau} = \frac{s}{v_1} \\ h_1 = \frac{1}{2} g t^2_{\pi\tau} \end{cases} \Leftrightarrow h_1 = \frac{1}{2} g \frac{s^2}{v_1^2} \Leftrightarrow 2v_1^2 h_1 = g s^2 \Leftrightarrow 2 \cdot 2g(H - h_1) h_1 = g s^2 \Leftrightarrow 4h_1(H - h_1) = s^2 \quad (2)$$

Σημείο Z

$$\begin{cases} x_z = \frac{s}{2} \\ y_z = h_1 - h_2 = h_1 - \frac{21H}{32} \end{cases} \Leftrightarrow \begin{cases} \text{όμως } x_z = u_1 \cdot t_1 \\ y_z = \frac{1}{2} g t_1^2 \end{cases} \Leftrightarrow y_z = \frac{1}{2} g \cdot \frac{x_z^2}{v_1^2} \Leftrightarrow 2v_1^2 \cdot Y_z = g \cdot x_z^2 \Leftrightarrow$$

$$\stackrel{(1)}{\Leftrightarrow} 2 \cdot 2g(H - h_1) \left(h_1 - \frac{21H}{32} \right) = g \cdot \frac{s^2}{4} \Leftrightarrow 16(H - h_1) \left(h_1 - \frac{21H}{32} \right) = s^2 \quad (3)$$

$$\text{Από (3), (2)} \Leftrightarrow 4h_1(H - h_1) = 16(H - h_1) \left(h_1 - \frac{21H}{32} \right) \Leftrightarrow h_1 = 4h_1 - 4 \frac{21H}{32} \Leftrightarrow \frac{21H}{8} = 3h_1 \Leftrightarrow h_1 = \frac{7H}{8}$$

$$\text{Τελικά } m_{\varepsilon\iota\sigma} = m_{\varepsilon\xi} \Leftrightarrow \frac{dm_{\varepsilon\iota\sigma}}{dt} = \frac{dm_{\varepsilon\xi}}{dt} \Leftrightarrow \rho \frac{dv_{\varepsilon\iota\sigma}}{dt} = \rho \frac{dv_{\varepsilon\xi}}{dt} \Leftrightarrow \Pi_{\varepsilon\iota\sigma} = \Pi_{\varepsilon\xi} \Leftrightarrow$$

$$\Pi = A \cdot v_1 = A \cdot \sqrt{2g(H - h_1)} = A \cdot \sqrt{2g \left(H - \frac{7H}{8} \right)} \Leftrightarrow \Pi = A \cdot \sqrt{2g \left(\frac{H}{8} \right)} \Leftrightarrow \Pi = \frac{A}{2} \sqrt{gH}$$

ΘΕΜΑ Γ

Λείπει σχήμα

$$E_{\varepsilon\pi} = -N \frac{d\varphi}{dt} \Leftrightarrow |E_{\varepsilon\pi}| = N \left| \frac{d\varphi}{dt} \right| \stackrel{\Phi \uparrow}{\underset{df/dt > 0}} \frac{d\varphi}{dt} = (B_1 \cdot l \cdot x)' = B_1 l \frac{dx}{dt} = Bvl$$

$$I_{\varepsilon\pi} = \frac{E_{\varepsilon\pi}}{R_{o\lambda}} = \frac{Bvl}{R_{o\lambda}}, F_i = B_i l \eta \mu \frac{\pi}{2} = \frac{B_1^2 v l^2}{R_{n\pi} + R_1}$$

$$\Sigma \vec{F} = m \cdot \vec{a} \Leftrightarrow |\vec{F}| - |\vec{F}_2| = m|\vec{a}| \Leftrightarrow |\vec{F}| - \frac{B_1^2 v l^2}{R_{n\pi} + R_1} = ma \quad \text{Ευθεία Μετακίνηση επιταχυνόμενη κίνηση με } a \downarrow \text{ καθώς } u \uparrow$$

$$\text{Για } a = 0 \text{ m/s}^2 \rightarrow v = v_{o\rho}$$

$$\text{Οπότε } |\vec{F}| = \frac{B_1^2 v l^2}{R_{n\pi} + R_1} \Leftrightarrow v_{o\rho} = \frac{|\vec{F}| (R_{n\pi} + R_1)}{B_1^2 l^2} \Leftrightarrow v_{o\rho} = \frac{\frac{0,8 \cdot (3+2)}{1 \cdot 1} m}{s} = \frac{4m}{s \Leftrightarrow v_{o\rho}} = 4 \text{ m/s}$$



Γ₂

$$t_1 \rightarrow \text{καταργείται η } \vec{F}, \vec{F}_2 = \vec{0} \rightarrow \vec{F}_{2x} \vec{0}$$

Ευθεία ομαλή κίνηση με $v_{0\rho} = 4\text{m/s}$

$$\text{Αρκεί } \Sigma \vec{F} = \vec{0} \Leftrightarrow |\vec{F}_1| = |\vec{F}_{2x}| \Leftrightarrow |\vec{F}'| = B_3 \frac{B_3 v_{0\rho} \cdot l}{R_{n\pi} + R_1} l = \frac{B_3^2 v_{0\rho} l^2}{R_{n\pi} + R_1} = \frac{1 \cdot 4 \cdot 1}{5} N \Leftrightarrow |\vec{F}'| = 0,8 N$$

Διεύθυνση $x'x$, φορά δεξιά

Γ₃

Νόμος Nenmann

$$|Q_{cn}| = N \left| \frac{\Delta \Phi}{Rg} \right| \Leftrightarrow \frac{B_3 \cdot l \cdot \Delta X}{R_{n\pi} + R_1} = |l_{cn}| \Leftrightarrow \Delta X = \frac{(R_{n\pi} + R_1) |Q_{cn}|}{B_3 \cdot l} = \frac{5 \cdot 0,2}{1 \cdot 1} = 1\text{m}$$

$$v_{0\rho} \text{ σταθ} \rightarrow i_{\varepsilon\pi} = \frac{B_3 v_{0\rho} \cdot l}{R_{n\pi} + R_1} = \frac{4}{5} A = 0,8 = \text{σταθ}$$

$$Q_{Ro\lambda} = i_{\varepsilon\pi}^2 \cdot (R_{n\pi} + R_1) \Delta t = i_{\varepsilon\pi}^2 \cdot (R_{n\pi} + R_1) \frac{\Delta x}{v_{0\rho}} =$$

$$= 0,64 \cdot 5 \frac{1}{4} j = 0,16 \cdot 5 j = 0,8 j$$

Γ₄

δ κλειστό

ισοδύναμη κίνηση

$$i = l_1 + l_2$$

$$V_{n\pi} = B_3 v l - i \cdot R_{n\pi} = i_1 R_1 = i_{1=2} R_2$$

$$R_1 // R_2 \rightarrow R_{1,2} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2 \cdot 2}{2 + 2} \Omega = 1 \Omega$$

$$R_{o\lambda} = R_{n\pi} + R_{1,2} = 4 \Omega$$

$$i_{o\lambda} = \frac{E_{\varepsilon\pi}}{R_{o\lambda}} = \frac{B_3 v_{0\rho} \cdot l}{R_{o\lambda}}, F_{1,3} = \frac{B^2 v_3 l^2}{R_{o\lambda}}$$

$$\text{Για } a' = 0 \text{ m/s}^2 \rightarrow v_{0\rho}'$$

$$\Sigma \vec{F} = m \cdot \vec{a} \Leftrightarrow |\vec{F}'| - |\vec{F}_{2,3}| = m |\vec{a}'| \Leftrightarrow |\vec{F}'| - \frac{B^2 v_3 l^2}{R_{o\lambda}} = m |\vec{a}'| \quad |\vec{a}'| \Leftrightarrow \text{m/s}^2 = v_{0\rho}'$$

$$\Leftrightarrow |\vec{F}'| = \frac{B_3^2 v_{0\rho}' l^2}{R_{o\lambda}} \Leftrightarrow v_{0\rho}' = \frac{R_{o\lambda} \cdot |\vec{F}'|}{B_3^2 l^2} v_{0\rho}' = 4 \cdot 0,8 \frac{\text{m}}{\text{s}} = 3,2 \text{m/s}$$

Επομένως

$$V_{o\lambda} = \frac{E_{\varepsilon\pi}}{R_2} = \frac{B_3 v_{0\rho}' \cdot l}{R_{o\lambda}} = \frac{1 \cdot 3,2 \cdot 1}{4} A = 0,8 A$$

Συνεπώς

$$V_{n\pi} = B_3 v_{0\rho}' \cdot l - i_{o\lambda} R_{n\pi} = 1 \cdot 3,2 \cdot 1 - 0,8 \cdot 3 = (3,2 - 2,4) v = 0,8 V$$

$$\text{Οπότε } i_1 = \frac{V_{n\pi}}{R_1} = \frac{0,8}{2} A = 0,4 A$$

$$i_2 = \frac{V_{n\pi}}{R_2} = \frac{0,8}{2} A = 0,4 A$$



ΘΕΜΑ Δ

Δ₁

$$\Sigma \vec{F} = \vec{0} \Leftrightarrow |\vec{T}_2| = m = m_2 g = 30 N$$

$$\Sigma \vec{\tau} = \vec{0} \Leftrightarrow \dots \Leftrightarrow T_1 \cdot r = T_2 \cdot 2r \Leftrightarrow T_1 = 2T_2 = 60 N$$

$$\text{Η ράβδος ισορροπεί } \Sigma \vec{z}_{(A)} = \vec{0} \Leftrightarrow \dots \Leftrightarrow$$

$$\Leftrightarrow + T_1 \cdot \frac{2l}{3} \eta \mu \theta - M g \cdot \frac{l}{2} \sigma \nu \nu 45^\circ + N_r \cdot l \eta \mu 45^\circ = 0$$

$$l \eta \mu 45^\circ = \sigma \nu \nu 45^\circ \quad N_r = \frac{M g}{2} - \frac{2 T_1}{3} \Leftrightarrow N_r = \left(50 - \frac{2 \cdot 60}{3} \right) N \Leftrightarrow N_r = 10 N$$

Δ₂

$$\Theta I_1 \rightarrow \Sigma \vec{F} = \vec{0} \Leftrightarrow K \Delta l_1 = m_1 g \eta \mu \varphi \Leftrightarrow \Delta l_1 = \frac{m_1 g \eta \mu \varphi}{K} = 0,05 m$$

$$\Theta I_{20\lambda} \rightarrow \Sigma \vec{F} = K \Delta l_2 = (m_1 + m_2) g \eta \mu \varphi \Leftrightarrow \Delta l_2 = \frac{(m_1 + m_2) g \eta \mu \varphi}{K} = 0,2 m$$

$$x_{\alpha\rho\chi} = -(\Delta l_2 - \Delta l_1) = -0,15 m = -\frac{15}{100} m = -\frac{3}{20} m$$

$$v_{\alpha\rho\chi} = v_K = \frac{5\sqrt{3}}{4} m/s$$

ΑΔΕΤ

$$K + v = E_l \Leftrightarrow \frac{1}{2} (m_1 + m_2) v_K^2 + \frac{1}{2} \Delta x^2_{\alpha\rho\chi} = \frac{1}{2} \Delta A^2 \Leftrightarrow \frac{1}{2} \Delta A^2 = \frac{1}{2} \frac{(m_1 + m_2)}{\Delta} v_K^2 + \frac{1}{2} \Delta x^2_{\alpha\rho\chi}$$

$$\Leftrightarrow A = \sqrt{\frac{m_1 + m_2}{K} \cdot v_K^2 + x^2_{\alpha\rho\chi}} \Leftrightarrow A = \sqrt{\frac{4}{100} \cdot \frac{9 \cdot 3}{16} + \frac{9}{400}} m \Leftrightarrow A = \sqrt{\frac{36}{400}} \Leftrightarrow A = \frac{6}{20} m \Leftrightarrow A = 0,3 m$$

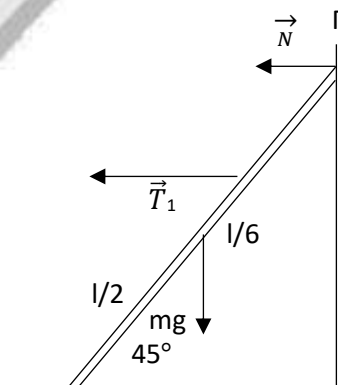
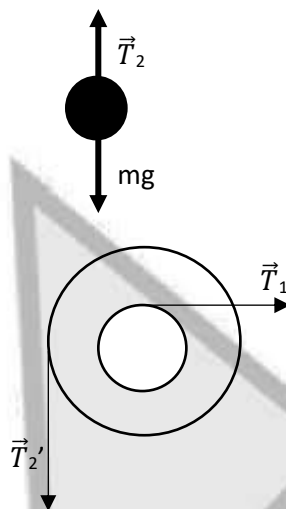
Δ₃

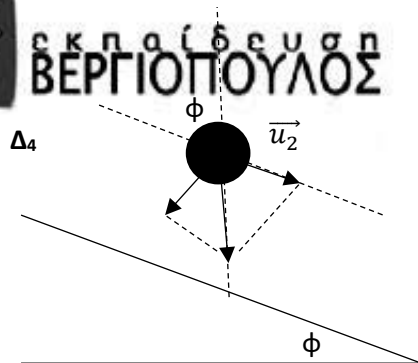
$$\left. \begin{array}{l} x(0) = -\frac{A}{2} \\ v(0) > 0 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} A \eta \mu \varphi_0 = -\frac{A}{2} \\ \omega A \sigma \nu \nu \varphi_0 > 0 \end{array} \right\} \Leftrightarrow$$

$$\Leftrightarrow \left. \begin{array}{l} \eta \mu \varphi_0 = -\frac{1}{2} \\ \sigma \nu \nu \varphi_0 > 0 \end{array} \right\} \Leftrightarrow \varphi_0 = 2\pi - \frac{\pi}{6} \Leftrightarrow \varphi_0 = \frac{11\pi}{6}$$

$$\Delta = K = (m_1 + m_2) \omega^2 \Leftrightarrow \omega = \sqrt{\frac{K}{m_1 + m_2}} = 5 \text{ rad/s}$$

$$x(+) = 0,3 \eta \mu \left(5 + -\frac{11\pi}{6} \right), (SI)$$





$$|v_{2y}| = |\vec{v}_2| \sigma \nu \nu \varphi$$

$$|\vec{v}_{2x}| = |\vec{v}_2| \eta \mu \varphi$$

ΑΔΟ x'x (H στο κεκλυμένο επίπεδο)

$$\dots \Leftrightarrow m_2 \vec{v}_2 \eta \mu \varphi = (m_1 + m_2) \vec{v}_K \Leftrightarrow |\vec{v}_2| = \frac{(m_1 + m_2) |\vec{v}_K|}{m_2 \eta \mu \varphi} = \frac{4 \frac{3\sqrt{3}}{4}}{3 \frac{1}{2}} = 2\sqrt{3} \text{ m/s}$$

$$\vec{v}_2 = \begin{cases} \text{μέτρο } 2\sqrt{3} \text{ m/s} \\ \text{διεύθυνση } y'y \\ \text{φορά στο σχήμα} \end{cases}$$